Fig. 2.

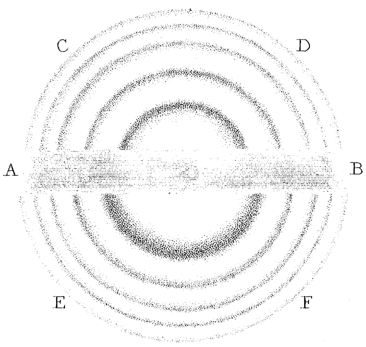


Fig. 3.

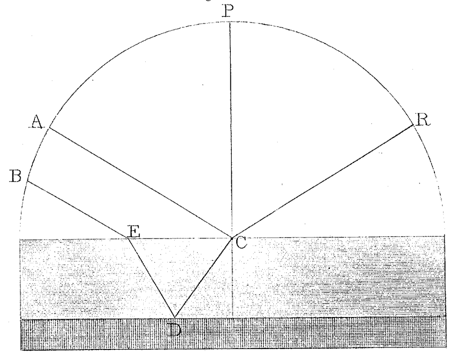


Fig. 1.

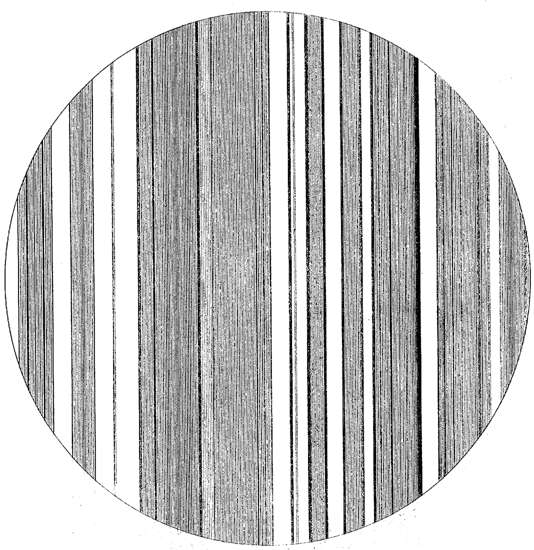
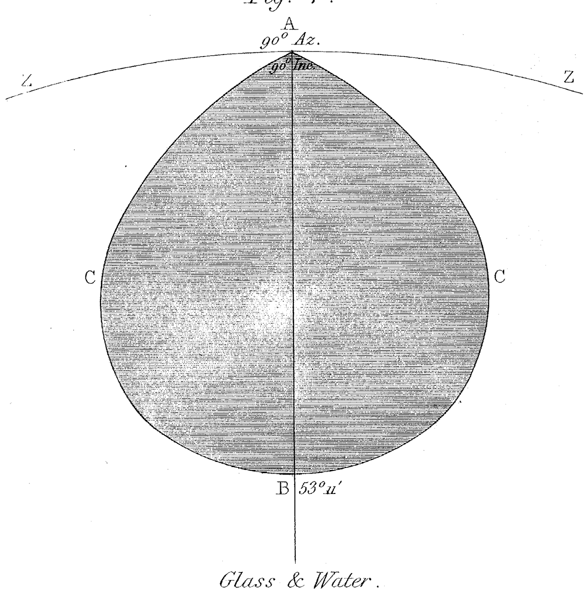
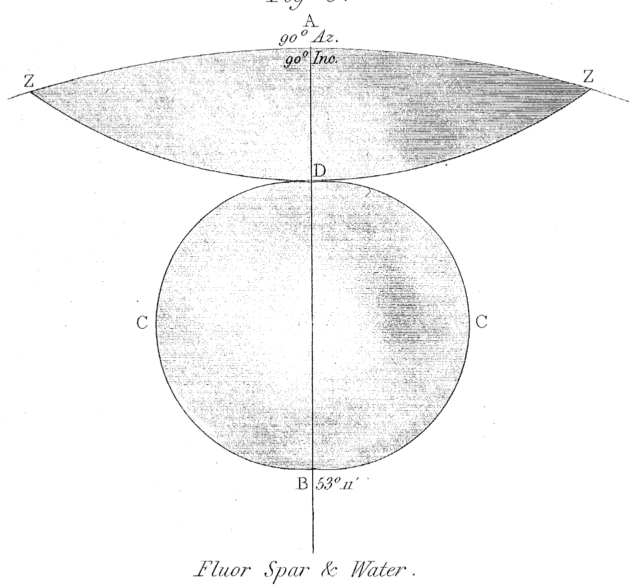


Fig. 4.



Glass & Water.

Fig 5.



Fluor Spar & Water.

J. Basire sc.

V. *On the Phenomena of Thin Plates of Solid and Fluid Substances exposed to Polarized Light.* By *Sir* David Brewster, *K.H. D.C.L. F.R.S. and V.P.R.S.Ed.*

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HAVING received from Dr. Joseph Reade one of his beautiful instruments called the *Iriscope*, and made several experiments with it, I soon perceived that it might be advantageously employed in various investigations in physical optics. This instrument consists mainly of a plate of highly polished black glass, having its surface smeared with a solution of fine soap, and subsequently dried by rubbing it clean with a piece of chamois leather. If we breathe upon the glass surface, thus prepared, through a glass tube, the vapour is deposited in brilliant coloured rings, the outermost of which is black, while the innermost has various colours, or no colour at all, in proportion to the quantity of vapour deposited. The colours in these rings, when seen by common light, correspond with Newton's *reflected rings*, or those which have *black centres*, the only difference being, that in the plate of vapour, which is thickest in the middle, the rings in the iriscope have black circumferences*. By using a large system of rings, or depositing the vapour in straight lines in the plane of incidence, we can at once observe the phenomena of the coloured rings or bands at various angles of incidence.

The first person who investigated the modification of Newton's rings in reference to polarized light was M. Arago, who has given an account of his observations in a beautiful and highly interesting memoir, in the third volume of the *Mémoires d'Arcueil*, published in 1817. Without knowing what had been done by M. Arago, Professor Airy entered upon the same inquiry in 1831 and 1832; but the phenomena which he observed were the same as those which had been previously discovered by M. Arago, with the exception of the modification of the rings when formed by a lens pressed against the surface of a diamond.

When Newton's rings are formed by a lens pressed against a surface of glass, M. Arago observed that they were black centred, as usual; and whether viewed with the eye or with a doubly refracting rhomb of Iceland spar, that the single or the

* These rings may be formed upon almost all transparent bodies with more or less brilliancy, though I have found several substances, and occasionally pieces of glass, that will not absorb the soap. The rings are produced upon natural as well as artificial surfaces, that is, upon transparent surfaces produced by fusion or crystallization, as well as upon those polished by art. The soap being gradually dissolved by the vapour, requires to be frequently renewed. I find that other substances, particularly some of the oils, produce the same effect as soap. The rings disappear quickly by evaporation, and their brilliancy and purity of colour depend on the relative temperature of the vapour and the glass.

double system of rings had the same colours and the same diameters, the rings being completely polarized at the polarizing angle of the glass.

When the lens, however, was pressed against a metallic mirror, and examined with a doubly refracting rhomboid, two images perfectly similar appeared between a perpendicular incidence, and that of 55° or the polarizing angle of glass. One of the images disappeared entirely at this angle of 55°, when the principal section of the rhomboid was perpendicular or parallel to the plane of reflexion; but reappeared at greater incidences, with this remarkable peculiarity, that the colour of each of the rings which composed it was complementary to that of the corresponding rings in the image which had disappeared.

M. Arago likewise remarks that we may easily perceive with the eye, naked and without the assistance of any crystal, that at a certain angle near 55° the rings are composed of two distinct sets having unequal diameters, the rhomboid separating in a great measure the two sets of rings, because they are very unequally polarized. He likewise found that these phenomena were not produced when the rings were formed upon *native sulphur* and *diamond.*

"If the presence of a metallic mirror," says M. Arago, "is necessary for the production of the phenomenon in question when the rings are formed upon a plate of air, the case is otherwise when the thin body has much more density, and is in contact by one of its faces with another medium of sufficient refractive power. Thus *coal* presents often in its cleavages very bright colours, produced by an extremely thin substance, and which are decomposed into two complementary images when they are examined with a rhomboid under sufficiently oblique incidences. The colours which are formed artificially by the progress of evaporation, on thin films of alcohol or oil of sassafras, deposited upon coal or any other analogous substance, give rise also to two images, dissimilar, and of opposite tints*."

In order to investigate the phenomena of the rings of vapour in the iriscope, I illuminated them with light polarized in an azimuth of 90°, or perpendicularly to the plane of incidence, and examined them by a magnifying glass, when the centre of the rings was seen by light reflected at about 53° 11′, the polarizing angle of water. The effect, which was very striking, is shown in Plate I. fig. 2. The central part, A B, of the system of rings, C D E F, was without rings and colours of any kind: the upper half, C D, was part of a system of rings with *white* circumferences, and was formed by polarized light incident on the film at an angle *greater* than the polarizing angle of water; while the under half, E F, was part of a system of rings with *black* circumferences like those seen by common light, and was formed by polarized light incident on the film at an angle *less* than the polarizing angle of water.

The absence of rings in the middle portion, A B, was of course owing to there being no light reflected from the *first* surface of the film with which that reflected from the *second* surface could interfere; and the reason of there being light reflected

* Mémoires de Physique et de Chimie de la Société d'Arcueil, tom. iii. p. 363. Paris, 1817.

from the second surface was, that the light reflected from it was not incident at its polarizing angle.

I have elsewhere shown*, that when a film of water is laid upon glass whose refractive index is above 1·508, there is no angle of incidence upon the first surface of the film which will allow the refracted ray to fall upon the glass at the polarizing angle; and hence at every angle of incidence on the film, the refracted light is reflected from the glass *at angles less than the polarizing angle of the united media*, or less than an angle whose tangent is equal to $\frac{m}{m'}$, m being the refractive index of the glass, and m' that of the water. When the refractive index of the glass is 1·508, the angle of incidence on the film must be 90° exactly, in order that the refracted ray may fall upon the glass at the polarizing angle whose tangent is equal to $\frac{m}{m'}$.

Now as the portion of the coloured rings at C D, fig. 2., is formed by the interference of two pencils, C A, D E B, fig. 3., one of which, C A, is reflected at an angle, P C A, *above* the polarizing angle of water, and the other, E B, at an angle *below* or *less* than that angle; while the portion E F, fig. 2., is formed by the interference of two pencils, which are both reflected at angles *below* or *less* than that angle, we may suppose that in the formation of the rings with a white circumference, analogous to those with a white centre, there is a loss of half an undulation, while that loss takes place in the interference of common light, or of two pencils reflected on the same side of the polarizing angle.

When the rings are seen at angles between 0° of incidence and 53° 11′, the polarizing angle of water, they are *black* in the circumference, like the portion shown at E F, fig. 2.; and when they are seen at incidences between 53° 11′ and 90°, they are *white* in the circumference, like the portion shown at C D, fig. 2.

If the rings of vapour are formed upon a polished surface of *fluor spar*, additional phenomena will be exhibited. At all incidences, from 0° to about 78°, rings of the same character will be seen as already described; but the ratio of the refractive powers of water and fluor spar is such, that at an incidence of 78° 4′ upon the surface of the vapour, the light incident on the spar will be reflected at the polarizing angle of the united media. Thus if $m = 1·437$, the refractive index of fluor spar, and $m' = 1·336$, the refractive index of water, then $\frac{m}{m'} = 1·0716$, the refractive index of the united media, or of their separating surface. The polarizing angle for this surface will therefore be an angle whose tangent is 1·0756 or 47° 5′, and the angle of incidence on the first surface of the watery film corresponding to the angle of refraction 47° 5′, which is the angle of incidence on the second surface, is 78°·4.

At an incidence of 78° 4′, therefore, the rings will disappear altogether, as at 53° 11′, because the pencil incident on the spar will not be reflected. At incidences greater

* Philosophical Transactions, 1815, p. 138.

than 78° 4′ the system of rings with the black circumference will again appear as at incidences below 53° 11′, and will be visible up to 90° of incidence, the interfering pencils being now both reflected at angles above the polarizing angle of the surfaces which reflect them.

This experiment with vapour and fluor spar I have not made; and it may be difficult to see the rings at such an oblique incidence. If the rings are formed by soap upon *plate glass*, or by *alcohol* upon *fluor spar*, the second disappearance of the rings may be seen:

$$\frac{\text{Plate glass}}{\text{soap}}\ \frac{m}{m'} = \frac{1{\cdot}510}{1{\cdot}487} = 1{\cdot}0154.$$

Polarizing angle at second surface of the soap . . . . 45° 26′
Angle of incidence on the first surface . . . . 71° 45′

$$\frac{\text{Fluor spar}}{\text{alcohol}}\ \frac{m}{m'} = \frac{1{\cdot}437}{1{\cdot}370} = 1{\cdot}049.$$

Polarizing angle at second surface of alcohol . . . . . 46° 22′
Angle of incidence on the first surface . . . . 82° 32′

If we call m, m' the indices of refraction of the two substances, viz. the *film* and the *surface* upon which it rests, m being the larger index, then a ray incident at 90° will fall upon the common surface of the two media at the polarizing angle of that surface, when the angle of refraction at the first surface is equal to the tangent, or cotangent of the polarizing angle, according as the refractive power of the film is less or greater than that of the body upon which it rests.

Hence we have $\sin i' = \frac{1}{m}$ or $\frac{1}{m'}$,

and

$$\tan i' = \frac{m}{m'}, \text{ or } \cot i' = \frac{m}{m'},$$

and

$$m = \frac{m'}{\sqrt{m'^2 - 1}}, \text{ and } m' = \frac{m}{\sqrt{m^2 - 1}},$$

when a ray incident at 90° is polarized at the second surface, or falls upon it at the polarizing angle.

These formulæ enable us to discover between what limits of refractive power the second disappearance of the rings can take place, and consequently what substances we should employ in order to observe it. In this manner we obtain the following results for the mean rays of the spectrum:—

Values of m'.	Values of $\frac{m'}{\sqrt{m'^2 - 1}}$, or m.
3·000	1·061
2·500	1·090
2·000	1·154
1·900	1·176

Values of m'.	Values of $\frac{m'}{\sqrt{m'^2 - 1}}$, or m.
1·800	1·202
1·700	1·236
1·600	1·281
1·554	1·307
1·508	1·336
1·500	1·341
1·400	1·428
1·336	1·508
1·307	1·554

The limits, therefore, between which the *second* disappearance of the rings can take place are 1·554, the index for *quartz* and *flint glass,* and 1·307, the index for *ice.* But though the range is very limited, it nevertheless includes a considerable variety of solid and fluid bodies. I have omitted the indices of TABASHEER, and the fluids produced by the compression of gaseous bodies, because, though their refractive powers are beneath 1·307, they cannot be used in the present inquiry.

When m and m' are thus related, the *white centred rings* will just disappear when $i = 90°$, the light being then incident on the second surface at its polarizing angle. But if we use a film of still less refractive power in relation to the second body, the refracted rays will fall on the second surface at an angle *greater* than the polarizing angle (i being still 90°), and consequently *the black centred rings will reappear,* and there will be some angle of incidence I on the film, less than 90°, at which the angle of refraction i' will be equal to the polarizing angle of the second surface. This angle will be found from the expression

$$\sin \mathrm{I} = \frac{m\,m'}{\sqrt{m^2 + m'^2}}.$$

When $m = m'$ no rings whatever will be formed, as no light is reflected at the common surface; but if $m = m'$ only for a particular colour in the spectrum of each substance, and if these indices differ considerably for another colour, rings will be formed in which that colour predominates, in which $m > m'$, or $m < m'$. This takes place in a remarkable manner with *oil of cassia* and *flint glass,* in which $m = m'$ for the *red* rays, but $m > m$ for the *blue* rays. The consequence of this is, that a quantity of *blue* light is reflected from the separating surface of the oil and the glass; and hence if a sufficiently thin film of oil of cassia is laid upon the glass, *blue* would greatly predominate in the system of rings.

Hitherto the azimuth of the polarized light has been 90°, or perpendicular to the plane of reflexion. Let us now suppose that its azimuth is gradually changed from 90° to 0° by the rotation of the polarizing surface or crystal.

At all azimuths, from 90° to 0°, the rings with the black circumference are seen, between the angles of 0° and 53° 11′, and at the incidence of 53° 11′. But at inci-

dences between 53° 11′ and 90°, in the case of the iriscope, very interesting phenomena appear. We shall first describe what takes place at 56° 45′, the polarizing angle of the black glass. At this angle none of the polarized light is reflected when the azimuth is 90°, and the rings with the *white* circumference are beautifully seen on the dark ground of the glass, which now reflects no light. As the azimuth is changed to 87°, 88°, &c., the black glass reflects a little light, and the two surfaces of the film a little more light, the rings gradually become fainter and fainter, till at an azimuth of about 79° 0′ they disappear exactly as they did at 53° 11′, and in the azimuth 90°. When this disappearance takes place, the light reflected from the glass seems to be exactly equal to the light reflected from both surfaces of the film. At other angles of incidence the rings disappeared at different azimuths, varying from 90° to about 45°, as the angle of incidence varied from 53° 11′ to 90°. I found it difficult, however, to measure these azimuths with any accuracy, as the rings were not permanent; and I was therefore obliged to form the colours of thin plates upon highly refracting substances, such as *diamond, chromate of lead, artificial realgar*, and *greenockite* (the most refractive of all bodies), which had high polarizing angles. A solution of fine soap gave brilliant colours when dried, and in this way I obtained the following results with the surface of a very fine diamond. The index of refraction of the soap was 1·475, and that of the diamond 2·44, and their respective polarizing angles 55° 52′, and 67° 43′.

Angle of Incidence of the Polarized Light.	Azimuth of the Plane of Polarization at which the rings disappear. Observed.	Calculated.
55° 52′	90° 0′	90° 0′
60	73 0	74 27
65	68 30	67 49
67 43	66 20	65 10
70	63 30	63 14
75	59 15	58 23
90		46 30

As the disappearance of the rings was not owing to the extinction of one of the interfering pencils, as at 55° 52′, for a sufficient quantity of polarized light was reflected from both surfaces of the film, there was reason to believe that it might arise from the two pencils being polarized at right angles to each other, in conformity with the law relating to the action of the second surfaces of plates which I have given in a former paper*.

Calling x the azimuth of primitive polarization, i the angle of incidence on the *first* surface of the film, i' the corresponding angle of refraction, and consequently the angle of incidence on the *second* surface, i'' the angle of refraction at the *second* surface,

* Philosophical Transactions, 1830, pp. 148, 149.

and φ = the inclination of the plane of polarization of the reflected pencil C A, fig. 3.

φ' = that of the refracted pencil C D,

φ'' = that of the reflected pencil D E, and

φ''' = that of the refracted pencil E B, with which C A interferes; then by FRESNEL's formula we have for the ray C A,

$$\tan \varphi = \tan x \,.\, \frac{\cos (i + i')}{\cos (i - i')};$$

and by my formulæ* we have

$$\cot \varphi' = \cot x \cos (i - i')$$

$$\tan \varphi' = \tan x \,.\, \frac{1}{\cos (i - i')}$$

$$\tan \varphi'' = \tan x' \,.\, \frac{\cos (i' + i'')}{\cos (i' - i'')}.$$

But, after one refraction,

$$\tan x' = \tan \varphi = \tan x \,.\, \frac{1}{\cos (i - i')};$$

hence

$$\tan \varphi'' = \tan x \,.\, \frac{1}{\cos (i - i')} \cdot \frac{\cos (i' + i'')}{\cos (i' - i'')}$$

and

$$\cot \varphi'' = \frac{1}{\tan x} \,.\, \cos (i - i') \,.\, \frac{\cos (i' - i'')}{\cos (i' + i'')}.$$

And multiplying this by $\cos (i - i')$ for the change of plane produced by the second refraction at E, we have for the ray E B,

$$\cot \varphi''' = \cot x \cos^2 (i - i') \,.\, \frac{\cos (i' - i'')}{\cos (i' + i'')}.$$

Now the two pencils which interfere, viz. C A and E B, have their planes of polarization inclined at angles φ and φ''' to the plane of reflexion; but in order that these angles may be complementary to each other, or may together make 90°, we must have $\tan \varphi = \cot \varphi'''$, or

$$\tan x \frac{\cos (i + i')}{\cos (i - i')} = \cot x \cos^2 (i - i') \cdot \frac{\cos (i' - i'')}{\cos (i' + i'')};$$

and consequently

$$\tan^2 x = \cos^2 (i - i') \cdot \frac{\cos (i - i')}{\cos (i + i')} \cdot \frac{\cos (i' + i'')}{\cos (i' - i'')};$$

and

$$\tan x = \cos (i - i') \,.\, \sqrt{\left(\frac{\cos (i - i')}{\cos (i + i')} \cdot \frac{\cos (i' - i'')}{\cos (i' + i'')}\right)}.$$

When the angle of incidence is 90°, $\cos (i + i') = \sin i'$, and $\cos (i - i') = \sin i'$, and hence

$$\tan x = \frac{1}{m} \sqrt{\frac{\cos (i' - i'')}{\cos (i' + i'')}}.$$

* Philosophical Transactions, 1830.

If we now calculate by these formulæ the values of x for the different angles of incidence in the preceding Table, and subtract them from 90°, we shall have the numbers in the third column of the Table, which agree with those observed within the limits of the errors of observation. In the case of *water* and *glass*, too, where the azimuth of disappearance was observed to be about 79° or 11°, the formula gives 79° 28′, or 10° 32′, at an incidence of 56° 45′.

In order to ascertain the relation between the mutual inclination of the planes of polarization of the interfering pencils when they produced *black-centred* or *white-centred* rings, I have computed the following Table for an incidence of 56° 45′.

Azimuth of Polarized Light.	$+\varphi$	$-\varphi'''$	Film of *water* and *glass*. Inclination of Planes φ and φ'''.	
° ′	° ′	° ′	° ′	
90	90 0	90 0	180 0	*White*-centred rings.
87 30	74 43	82 45	157 28	
85 0	49 30	75 4	124 34	
79 28	28 26	61 34	90 0	*No rings.*
70 0	15 28	43 19	58 47	*Black*-centred rings.
45 0	5 45	18 57	24 42	
35 0	4 3	13 3	17 6	
20 0	2 6	7 7	9 13	
0 0	0 0	0 0	0 0	

By taking φ *positive*, or on the *right*-hand side of the plane of reflexion, then φ''' must be *negative*, or on the left-hand side of that plane*; hence $+\varphi$, $-\varphi'''$ will be the mutual inclinations of the planes of polarization of the interfering pencils, and we obtain the important law,

That when two polarized pencils reflected from the surfaces of a thin plate lying on a reflecting surface of a different refractive power interfere, half an undulation is not lost, and WHITE-*centred rings are produced, provided the mutual inclination of their planes of polarization is greater than* 90°; *and that when this inclination is less than* 90°, *half an undulation is lost, and* BLACK-*centred rings are produced; when the inclination is exactly* 90°, *the pencils do not interfere, and no rings are produced.*

At an incidence of 45° upon water and glass, where the signs of φ and φ''' are the same, the maximum difference in the planes of polarization is 23° 12′, which takes place in azimuth 70° 30′; and at an incidence of 10° the greatest difference is 2° 16′, which takes place at an azimuth of about 45°.

In the case of *soap* and *plate glass*, where the black-centred rings appear beyond the incidence of 71° 45′, the difference of inclination in the planes of the two pencils is also less than 90°.

I was now desirous of examining the phenomena of a perfect system of rings when the film had a greater refractive power than the substance upon which it was laid;

* See Philosophical Transactions, 1830, p. 70, fig. 1.

after many ineffectual attempts to obtain such a system, I succeeded by laying a very small portion of *oil of laurel* upon *water* placed in a black vessel, or on the surface of diluted or real ink. The rings thus produced are splendid beyond description, and exhibit the various phenomena with singular beauty. As the polarizing angle of the oil *exceeds* that of the water, the *black*-centred rings are seen at the polarizing angle of the water, when the reflected light disappears. They continue to be seen till we reach the polarizing angle of the oil, when the rings disappear, and the white-centred ones commence, and continue till we reach the incidence of 90°*.

In forming thin films upon metallic surfaces, I employed many of the metals, and found the phenomena nearly the same upon them all, and differing very little from those produced upon transparent bodies. On a fine specimen of *specular iron ore*, I found a system of rings ready formed, with three orders of colours. The azimuth of the polarized light being inclined 90° to the plane of reflexion, the system of rings disappeared wholly at an angle of incidence of 58° 36′, which is therefore the polarizing angle of the unknown substance of which it was formed: consequently its index of refraction is about 1·638. Between this angle and 90° of incidence, the *white*-centred rings appeared; but at 72° 39′, the polarizing angle of the iron (which gives its refractive power for the *red* rays 3·200), the rings were singularly fine, being seen on a beautiful blue ground, produced by the disappearance of the *red* light, which is polarized at that angle. I now measured the azimuth of the plane of polarization when the rings disappeared, which was 59° 25′, whereas by the formula it is 57° 59′; a discrepancy not to be wondered at, when we consider that the index of refraction for the red rays, viz. 3·200, was used, in place of that for the mean ray, which is not known. The inclination of the planes of polarization of the two interfering pencils, when calculated by the previous formulæ, is + 32° 7′, and − 57° 53′; so that these planes being inclined 90° to each other, as in the case of soap and diamond, no interference takes place, and the rings disappear.

In the fine specimens of *oligist iron ore* from Elba, I have found crystals covered with the most beautiful coloured films, both of uniform and variable thickness. These films are not acted upon by the ordinary acids, like the coloured films upon steel, and appear, from their optical properties, to be of a metallic nature. When they are exposed to a polarized ray, they exhibit generally the same phenomena as the films already described; but there is no angle of incidence at which the colours disappear, either in the azimuth of 90°, at the polarizing angle of the first surface of the film,

* These thin plates of oil of laurel exhibit some curious phenomena, which I believe have not been noticed. If we wet with water, alcohol, or the oil of laurel itself, the extremity of a short piece of wire, such as a large pin, and hold the pin in the hand, so that its head may be above, and almost touching the film, the film will recede in little waves of a circular shape, which form a new system of coloured rings; and they become covered with the vapour from the fluid on the head of the pin in such small particles that they reflect no light, and the rings appear to be blackened. By withdrawing the pin, the film is restored to its former state. The same effect is produced by heating the pin, or the fluid upon it, to promote evaporation.

or in those azimuths where the pencils, from the first and second surface, have their planes of polarization inclined 90° to each other. This, no doubt, arises from the high dispersive power of the film, in consequence of which the different homogeneous rays are polarized at angles considerably different from each other.

The phenomena of transparent films of low refractive power, when laid upon the polished surfaces of metals, and exposed to polarized light, are not very different from those which are exhibited when the film rests upon a transparent surface. I at first used a solution of soap, which produced pretty good tints on speculum metal; but at last I fell upon a method of laying down the most beautiful systems of coloured rings upon all surfaces of all forms, whether metallic, transparent, or opake. For this purpose I used the *oil of laurel*, which, when placed upon the surface of water, expands into a film, which gives the finest system of coloured rings. Having laid the plate of polished metal in a small porous wooden tray, such as is used for holding minerals, I poured water into it, so as to cover the metallic surface to the depth of the 50th part of an inch. I then formed a film of the oil upon the water, immediately above the metallic surface. In a short time the absorption of the water by the porous tray allowed the film of oil to descend and rest upon the metallic surface*. When the adhering moisture was removed by evaporation, the film was extremely beautiful; and if protected from dust may be preserved for any length of time.

Having laid a film of this kind upon *speculum metal*, I obtained the following results. The coloured rings disappeared almost completely at 56°, the polarizing angle of the oil. The *black-centred* rings appeared at all angles less than 56°, and the *white-centred* rings at all angles above it. Both the systems of rings were exceedingly distinct at the greatest angles of incidence, whereas on transparent surfaces of low refractive power, they can scarcely be seen at such angles. When the azimuth of the polarized ray varies from 90° to 0°, the rings disappear at different angles of incidence; or when the angles of incidence vary, the rings disappear in different azimuths. I measured these azimuths when the polarized ray was incident upon speculum metal, and obtained the following numbers:—

Angles of Incidence.	Azimuth in which the rings disappear. Observed.	Calculated.	Difference.
° ′	° ′	° ′	
90		40 23	
71 50	56 25	57 22	− 0 57
60 0	65 45	65 4	+ 0 41
56 5	90 0	90 0	

In computing column third from the formula in p. 49, I used 1·49 as the index of refraction of *oil of laurel*, and 4·011 as the index of refraction for *speculum metal*, as deduced from my experiments on its elliptic polarization†.

* The same effect is produced more slowly by evaporation; or the water may be sucked out of the tray by a tube, or run off by an aperture.

† Philosophical Transactions, 1830, p. 324.

I have made similar experiments when the rings were transferred to *silver,* whose elliptical polarization approaches nearest to circular polarization ; and to *grain tin,* which appears to have the highest refractive power of any of the metals ; but I found it very difficult to ascertain with any accuracy the azimuths in which the rings disappear.

If we use common in place of polarized light in the preceding experiments, and analyse the reflected light by a rhomb of calcareous spar, the very same phenomena will be exhibited.

When the films or thin plates are not laid upon the surfaces of fluid or solid bodies, the phenomena are of an entirely different kind. At all angles of incidence, and in all azimuths, the colours and character of the rings are the same, whether we use common or polarized light. In obtaining this result I stretched thin films of various oils, such as *oil of laurel, oil of cassia, oil of turpentine,* and many others, across circular apertures, and examined them in light polarized in different azimuths. The rings of course vanished at the polarizing angle of the oil, and the brilliancy of the colours varied with the angles of azimuth and incidence, but the complementary rings never appeared, the rings being always those with the black centre*.

In order to understand the cause of this, we must inquire into the state of polarization of the interfering pencils. The ratio of refraction being the same at both surfaces of the film, we have

$$\tan \varphi = \tan x \, . \, \frac{\cos (i + i')}{\cos (i - i')}, \text{ and } \cot \varphi''' = \cot x \, . \, \frac{\cos^3 (i - i')}{\cos \ (i + i')} \, ;$$

and when $\tan \varphi = \cot \varphi'''$, which is the case when $\varphi + \varphi''' = 90°$, we have

$$\tan x = \frac{\cos^2 (i - i')}{\cos (i + i')}.$$

When $i = 90°$, $\tan \varphi = A$, or the azimuth of the polarized ray, and $\cot \varphi''' = \frac{\cos^3 i'}{\sin \ i'}$.

If we now compute the values of φ and φ''' at different angles of incidence and in different azimuths of the polarized light, we shall obtain the results in the following Table. In azimuths 0° and 90°, φ and $\varphi''' = 0$.

* The physical phenomena exhibited in these attenuated films are very remarkable. A current of fluid is projected from the margin and centre of the ring of fluid across the fluid surface, resembling the top of a pine apple. This movement makes the film thinner at some places than others, and hence arises an irregular system of coloured bands, with an incessant play of varying tints, as if the fluid were animated. The bands of colour are serrated with salient points, from which the fluid seems to shoot across the film. In the oils of cinnamon, naphtha, spearmint, wormwood, rapeseed, nutmegs, bergamot, savine, rosemary, &c., the phenomena are peculiarly beautiful. With poppy oil, the *red* and *green* tints of the 4th, 5th, and 6th orders were also seen.

Inclination of the planes of polarization of the two pencils, ϕ and ϕ'''.

Angles of Incidence.	Azimuth 22° 30′.		Azimuth 45°.		Azimuth 67° 30′.		Azimuth 80°.	
	Pencil from *first* surface.	Pencil from *second* surface.	Pencil from *first* surface.	Pencil from *second* surface.	Pencil from *first* surface.	Pencil from *second* surface.	Pencil from *first* surface.	Pencil from *second* surface.
	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
0	22 30	22 30	45 0	45 0	67 30	67 30	80 0	80 0
10	21 42	22 5	43 51	44 24	66 40	67 4	79 36	79 48
20	19 11	19 34	40 13	40 38	64 13	64 14	78 13	78 23
30	15 25	15 55	33 40	34 33	58 7	58 58	75 10	75 38
40	10 18	11 1	23 41	25 11	43 21	48 37	68 6	68 6
50	4 18	4 52	10 18	11 37	23 41	26 24	45 52	49 23
56 in 45	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0
60	2 6	2 35	5 4	6 13	12 5	14 44	26 42	31 42
70	7 54	11 52	18 32	26 53	39 0	50 45	62 16	70 49
80	15 11	24 41	33 13	47 58	57 41	69 32	74 56	80 58
85	18 40	33 34	39 12	58 2	63 5	75 30	77 48	83 43
90	22 30	43 57	45 0	66 44	67 30	79 54	80 0	85 40

The results in this Table, which may be considered as those of observation*, exhibit at one glance the general phenomena at all angles of incidence and azimuth.

The two interfering pencils are in every case reflected at angles either *both above* or *both below* the polarizing angle, and hence their planes of polarization are always on the same side of the plane of reflexion and in the same quadrant, and consequently they never can be at right angles to each other so as to prevent interference. For the same reason the inclination of the planes never can exceed 90°, so as to produce the complementary white-centred rings, in conformity with the law previously given.

If, for example, we compute the value of x in the preceding formula at an incidence of 70°, we shall find it 66° 25′, at which azimuth the inclinations φ and φ''' of the planes of polarization are 40° 47′, and 49° 53′; but though the sum of these angles is 90°, yet the real inclination of the planes is $\varphi''' - \varphi = 9° \; 6'$.

This property of parallel transparent films, of giving by reflexion pencils polarized in planes at various inclinations, when the incident light is polarized in different azimuths, enables us to obtain two pencils of polarized light, inclined at any angle, varying from 0° to 21° 44′ in glass, and to study the phenomena which such pencils exhibit, either in their mutual action, or in their relations to other properties of light.

But the phenomena become more varied and interesting when the second surface of the plate is *inclined* to the first. In this way we may produce effects analogous to those produced by a change in the refractive power of the second surface by contact with another refracting surface, and obtain pencils inclined 90° to each other, and therefore exhibiting the white-centred rings. The phenomena will in this case resemble those of a film of oil upon water.

When the refractive index of a parallel film exceeds 1·508, the ray is incident on the second surface at an angle less than the polarizing angle; but by inclining the

* See Philosophical Transactions, 1830, pp. 74, 138.

second surface we can make it fall upon it at a greater angle than the polarizing angle. The phenomena may be still more varied by inclining the surface of emergence to the surface of incidence*; but as it is not easy to obtain films with faces suitably inclined to each other, it is unnecessary to pursue this branch of the subject any further.

Such are the phenomena of *thin* and *thick* plates when viewed by polarized light, or by common light subsequently analysed by a doubly refracting rhomb. But if we use polarized light, and subsequently analyse the light transmitted through the thin plates, we shall obtain a series of very interesting and instructive phenomena, analogous to those produced by plates of doubly refracting crystals which exhibit the polarized tints. In both these cases, the film is interposed between a polarizing plate and an analyzing rhomb. If the film is too thick to produce colours, it will depolarize the polarized ray, in a manner analogous to that of a crystallized plate, which is not thin enough to give the polarized tints; and if the film is sufficiently thin to produce uniform tints, a coloured band or system of rings, with black or white centres. Their action is analogous to that of thin crystallized plates, which either produce uniform tints like the laminæ of sulphate of lime, or uniaxal or biaxal systems of rings.

It would be unprofitable to describe minutely the great variety of phenomena which thin plates thus exhibit, as they vary with the refractive power of the fluid or solid upon which they are laid, so that I shall confine myself to the case in which a thin plate of oil of laurel rests on the surface of a specimen of *artificial realgar*. In common light, the colours of this film are very beautiful, but when examined in polarized light by an analyzing rhomb, they are brilliant beyond description.

1. *When the azimuth of the polarized light* is 90°, and the incidence of the polarized ray 56° 5′, the polarizing angle of oil of laurel.

When the film is viewed without the polarizing rhomb, no rings are seen, as there is no light reflected from the first surface of the film, and consequently no interference.

When the film is viewed with the polarizing rhomb, having its principal section in the plane of incidence, no rings appear, either in its ordinary or extraordinary image. But if the plane of polarization is less or more than 90°, by even a small quantity, then after the rhomb has been turned round nearly 90° towards the right, a system of *black-centred* rings is seen for an instant, and these, after disappearing, are followed by a system of *white-centred* ones, the white-centred rings appearing first if the rhomb is turned to the left. The same phenomena are repeated in every quadrant of the circular motion of the rhomb.

2. *When the azimuth of the polarized light varies* from 90° to 0°, the incidence, being 56° 5′, as before.

At 90° azimuth the phenomena are as above described.

* Philosophical Transactions, 1830, p. 147, fig. 3.

At $67\frac{1}{2}°$. Rhomb 0°, no rings.

Rhomb turning to the right, the white-centred rings appear, then vanish, when the azimuth of the rhomb is less than $67\frac{1}{2}°$; then black-centred rings appear, which vanish at 180°; then succeed the white-centred ones, which vanish at about 210°; then the black-centred, which continue to 360°.

At 45°, $22\frac{1}{2}°$. The very same phenomena appear at these and other azimuths, the azimuths of the rhomb at which the rings disappear out of the plane of incidence being a little less than the azimuths of the polarized light.

At 0°. The evanescence of the rings takes place when the azimuths of the rhomb are 0°, 90°, 180°, and 270°, the *white-centred* rings appearing in the *first* and *third*, and the *black-centred* ones in the *second* and *fourth* quadrant.

3. *Azimuth of polarized light* 90°.

Incidence of polarized light 68° 3′, the polarizing angle of realgar.

At this angle all the light reflected from the realgar has disappeared, excepting a dark bluish purple, in the middle of which is seen, without using the rhomb, a splendid system of richly-coloured rings, with a *white centre*. When the rhomb is applied as before, and performs a complete revolution, the white-centred rings are seen all round, disappearing at 90° and 270°.

4. *When the azimuth of the polarized light varies from* 90° to 0°, the incidence being 68° 3′, as before.

At 90° azimuth, the phenomena are as above described.

At 80°, and all other azimuths, the *white-centred rings* are seen when the rhomb is at 0°; but they disappear at azimuths of the rhomb a little less than the azimuths of polarization, and are then succeeded by the *black-centred rings*.

At 0° azimuth, the rings disappear when the rhomb is at 0° and 180°, and are *black-centred* all round.

Without using the rhomb, the rings always disappear at the azimuth x, at which the planes of polarization of the interfering pencils are rectangular.

At incidences above 68° 3′, the phenomena are of the same character. The rings are *white-centred* in 90° of azimuth, and when the rhomb is at 0°. They become very brilliant about 45°. Near 90° of rotation the rings vanish, and immediately the black-centred system appears, which quickly vanishes, and is succeeded by the white-centred system.

5. *Angles of incidence less than* 56° 5′.

In 90° of azimuth of the polarized ray, and the rhomb being at 0°, the black-centred rings are seen, and continue to be seen during a complete revolution of the rhomb. In all azimuths, from 90° to 0°, the rings disappear by turning the rhomb to the left, the arch diminishing from 90° to 0°; but in azimuths of an intermediate magnitude, the disappearance of the rings is followed by the appearance of the *white-centred system*, which quickly disappears, and is succeeded by the *black-centred system*. This phenomenon is seen best near 45° of azimuth.

When the plates or films are too thick to give the coloured rings, the phenomena of the differently polarized pencils may be finely seen by using *coloured glasses*, in which the pencils reflected from both surfaces may be observed. If the glass is *green*, for example, the pencil or image of a small aperture or luminous body will be *green*, while that reflected from the first surface, though in reality colourless, will appear *red*, from the physiological action of the green light upon the retina. Hence the two differently polarized pencils will have different colours, as if they were the tints of polarized light. If these coloured glasses are laid upon, or cemented on one side to, metals or highly refracting substances, the polarization of the coloured pencils which they reflect will be modified according to the principles already explained, and they will exhibit many interesting phenomena, varying with the colours of the glasses, as if the colours were produced by the absorption of polarized light.

In order to convey a general idea of the different classes of phenomena described in the preceding paper, I have represented two of the most important in figs. 4 and 5.

1. *Glass and Water.*—When a film of aqueous vapour is laid upon glass whose index of refraction is 1·508, the rings disappear at 53° 11′, the polarizing angle of the water, and also in the various azimuths where the two interfering pencils are polarized in planes at right angles to each other. At all azimuths greater than these, and at angles of incidence above the polarizing angle, the *white-centred* rings appear; and at all azimuths less than these, and at all incidences (except those at which the white-centred rings are seen), the *black*-centred rings appear.

The following Table shows the values of x, or the azimuths of disappearance of the rings, as computed from the formula in p. 49:—

Angles of Incidence.	Azimuths.	Complements.
53° 11′	90° 0′	0° 0′
55 0	82 8	7 52
60 0	76 52	13 8
65	75 15	14 45
67	75 10	14 50
70	75 30	14 30
73	76 18	13 42
74	76 42	13 18
75	77 9	12 51
76	77 36	12 24
80	80 0	10 0
85	84 15	5 45
90	90 0	0 0

If we now conceive A B, fig. 4, to be the section of the plane of incidence, having the different incidences marked upon it from 90° to 53° 11′, and if round a centre in A B prolonged, where 0° of incidence falls, we describe the azimuthal circle Z A Z,

then the complements of the azimuths of the polarized light being set off from the corresponding angles of incidence on each side of A B, the curves A C B, A C B passing through these points will show at what angles of incidence and azimuth the rings disappear, in consequence of the planes of polarization of the two pencils being at these places rectangular.

At all incidences, and in all azimuths within the shaded space A C B C, the *white-centred rings* are seen, and at all other azimuths and incidences the *black-centred* rings are seen.

2. *Fluor Spar and Water.*—I have taken this combination as a specimen of the phenomena which take place at some incidences less than 90°, when the refracted ray falls on the second surface of the film, at angles greater than its polarizing angle. The following Table shows the values of x and their complements:—

Angles of Incidence.	Azimuths.	Complements.
° ′	° ′	° ′
53 11	0 0	0 0
55	82 35	7 25
60	77 47	12 13
63	76 54	13 6
65	76 41	13 19
67	77 6	12 54
70	78 9	11 51
75	82 0	8 0
78	88 41	11 9
78 4	90 0	0 0
80	83 28	6 32
85	77 31	12 29
90	74 14	15 46

By projecting these values, as is done in fig. 5, we obtain a double set of curves which unite at D, where the angle of incidence is 78° 4′, at which the refracted ray falls upon the second surface at its polarizing angle.

At all incidences, and in all azimuths within the shaded portions of the figure Z A Z D, D C B C, the *white-centred rings* are seen. At all azimuths and incidences corresponding with the outlines of the curves Z D Z, D C B C, the *rings disappear*; and at all azimuths and incidences without the shaded portions of the figure, the *black-centred rings* are seen*.

* No reference is made in these figures to the phenomena which are seen by using both polarized light and the analyzing rhomb.

St. Leonard's, St. Andrew's,
April 8, 1841.